Two Examples of Pattern-formation

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The emergence of spatial patterns in nature [1] continues to intrigue scientists from a broad range of natural sciences. Two recent examples will be presented during this talk.

The first part will present recent results on pattern-formation in dipolar Bose-Einstein condensates. In this system the possibility of obtaining quantum states with self-organized long-ranged ordering can be facilitated by quantum fluctuations which can suppress collapse and thereby pave the way for supersolids in this system [2]. Here, supersolidity refers to a state of matter which displays long-range ordering whilst maintaining a large superfluid fraction. I will present recent results which focus on the critical behaviour of the superfluid-supersolid phase-transition [3] and the crucial role quantum fluctuations can play for the latter. We find that quantum fluctuations can alter the order of the phase transition from first- to second order. Furthermore, apart from the usual triangular lattice of density droplets, quantum fluctuations can give rise to a novel quantum state whose density distribution displays a honeycomb structure [see Fig.1(a-e)].

The second part of this talk will focus on organising centers of three-dimensional excitable waves [4] and threedimensional pattern formation. There are a range of chemical, physical and biological excitable media that support spiral wave vortices. Examples include the Belousov-Zhabotinsky (BZ) redox reaction, the chemotaxis of slime mould and action potentials in cardiac tissue. Usually interacting vortex strings are prone to reconnection and untying. However, in excitable media such vortex strings can display remarkable topology preservation [5]. After presenting examples for their dynamics and their possible extraordinary robustness of topology [Fig.1 (f)] I will show recent theoretical and experimental results [6] realising vortex or scroll rings threaded by two counter-rotating filaments in the BZ reaction and discuss their dynamics [Fig. (g-i)].



FIG. 1. Isodensity surfaces of the three possible groundstates of a dipolar Bose-Einstein condensate (a-c). The superfluidsupersolid phase transition can be of first- (d) or, surprisingly, of second order (e) as illustrated by the dependence of ground state energy on the modulation amplitude A of the symmetry-broken state. (f) Shows a complex configuration of excitable spiral waves. The vortex line will untangle to a scroll or vortex ring upon evolution. (h) Experimental realisation, (i) numerical comparison and (g) three-dimensional visualisation of a complex vortex pattern of spiral waves in the BZ reaction.

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